Problem Set #3: Building and Applying the IS – LM
Econ 100B: Intermediate Macroeconomics

1) Assume the following model of the economy:

\[ Y = C + I + G \]
\[ C = 120 + 0.5(Y - T) \]
\[ I = 100 - 10r \]
\[ G = 50 \]
\[ T = 40 \]
\[ \left( \frac{M}{P} \right)^d = Y - 20r \]
\[ M^s = 600 \]
\[ P = 2 \]

a. Identify each of the variables and briefly explain their meaning.

- The variable \( Y \) represents real output or real income. From Chapter 2, we know that the value of the produced goods and services (real output) has to be equal to the value of the income earned in producing the goods and services (real income). The variable \( C \) represents the consumption of goods and services. The variable \( I \) represents investment by the firms. When firms purchase new capital goods, this counts as investment. When firms experience a change in their inventories, this also counts in the investment category of GDP. The variable \( G \) represents the government’s spending on newly produced goods and services. The variable \( T \) represents lump sum taxes, and \( Y - T \) represents disposable income. The variable \( M \) represents the nominal money supply, \( P \) is the price level, and \( M/P \) is the real money supply. The variable \( r \) is the real interest rate. The variable \( (M/P)^d \) represents real money demand. Consumption depends positively on disposable income, investment depends negatively on the real interest rate, and real money demand depends positively on real income and negatively on the real interest rate.

b. Derive the equation for the IS curve, showing \( Y \) as a function of \( r \) alone.

- The IS curve represents all combinations of the real interest rate \( r \) and real output \( Y \) such that the goods market is in equilibrium. The equation for the IS curve can be derived as follows:

\[ Y = C + I + G \]
\[ Y = (120 + 0.5(Y - T)) + (100 - 10r) + 50 \]
\[ Y = (120 + 0.5(Y - 40)) + (100 - 10r) + 50 \]
\[ Y = 250 + 0.5Y - 10r \]
\[ 0.5Y = 250 - 10r \]
\[ Y = 500 - 20r \]

c. Derive the equation for the LM curve, showing \( Y \) as a function of \( r \) alone.

- The LM curve represents all combinations of the real interest rate \( r \) and real output \( Y \) such that the money market is in equilibrium. The equation for the LM curve can be derived as follows:

\[ \left( \frac{M}{P} \right)^d = \left( \frac{M}{P} \right) \]
\[ Y - 20r = \frac{600}{2} \]
\[ Y = 300 + 20r \]

d. Graph both the IS and the LM curves.
e. What are the equilibrium level of income and equilibrium interest rate?

- $Y = 400$, $r = 5$

2) Suppose that the money demand function is $(\frac{M}{P})^d = 1,000 - 100r$, where $r$ is the interest rate in percent. The money supply $M$ is 1,000 and the price level $P$ is 2.

a. Graph the supply and demand for real money balances

- The downward sloping line in Figure 11-11 represents the money demand function $(M/P)d = 1,000 - 100r$. With $M = 1,000$ and $P = 2$, the real money supply $(M/P)^s = 500$. The real money supply is independent of the interest rate and is, therefore, represented by the vertical line in Figure 11-11.

b. What is the equilibrium interest rate?

- We can solve for the equilibrium interest rate by setting the supply and demand for real balances equal to each other:

$$500 = 1,000 - 100r$$
\[ r = 5 \]
- Therefore, the equilibrium real interest rate equals 5 percent.

c. Assume that the price level is fixed. What happens to the equilibrium interest rate if the supply of money is raised from 1,000 to 1,200?
- If the price level remains fixed at 2 and the supply of money is raised from 1,000 to 1,200, then the new supply of real balances \((M/P)^s\) equals 600. We can solve for the new equilibrium interest rate by setting the new \((M/P)^s\) equal to \((M/P)^d\):
\[
600 = 1,000 - 100r \\
100r = 400 \\
r = 4
\]
- Thus, increasing the money supply from 1,000 to 1,200 causes the equilibrium interest rate to fall from 5 percent to 4 percent.

d. If the Fed wishes to raise the interest rate to 7 percent, what money supply should it set?
- To determine at what level the Fed should set the money supply to raise the interest rate to 7 percent, set \((M/P)^s\) equal to \((M/P)^d\): \(M/P = 1,000 - 100r\). Setting the price level at 2 and substituting \(r = 7\), we find: \(M/2 = 1,000 - 100 \times 7\) or \(M = 600\). For the Fed to raise the interest rate from 5 percent to 7 percent, it must reduce the nominal money supply from 1,000 to 600.

3) Use the IS-LM model to predict the short-run effects of each of the following shocks on income. In each case, explain what the Fed should do to keep income at its initial level.

a. After the invention of a new high speed computer chip, may firms decide to upgrade their computer systems.
- The invention of the new high-speed chip increases investment demand, meaning that at every interest rate, firms want to invest more. The increase in the demand for investment goods shifts the IS curve out and to the right, raising income and employment. Figure 12-8 shows the effect graphically.

The increase in income from the higher investment demand also raises interest rates. This happens because the higher income raises demand for money; since the supply of money does not change, the interest rate must rise in order to restore equilibrium in the money market. The rise in interest rates partially offsets the increase in investment demand, so that output does not rise by the full
amount of the rightward shift in the IS curve. Overall, income, interest rates, consumption, and
investment all rise. If the Federal Reserve wants to keep output constant, then it must decrease
the money supply and increase interest rates further in order to offset the effect of the increase in
investment demand. When the Fed decreases the money supply, the LM curve will shift up and to
the left. Output will remain at the same level and the interest rate will be higher. There will be
no change in consumption and no change in investment. The interest rate will increase by enough
to completely offset the initial increase in investment demand.

b. A wave of credit card fraud increases the frequency by which people make transactions in cash.
– The increased demand for cash shifts the LM curve up. This happens because at any given level of
income and money supply, the interest rate necessary to equilibrate the money market is higher.
Figure 12-9 shows the effect of this LM shift graphically.

![](image)

– The upward shift in the LM curve lowers income and raises the interest rate. Consumption falls
because income falls, and investment falls because the interest rate rises due to the increase in
money demand. If the Federal Reserve wants to keep output constant, then they must increase
the money supply in order to lower the interest rate and bring output back to its original level.
The LM curve will shift down and to the right and return to its old position. In this case, nothing
will change.

c. A best-seller titled Retire Rich convinces the public to increase the percentage of their income devoted
to savings.
– At any given level of income, consumers now wish to save more and consume less. Because of this
downward shift in the consumption function, the IS curve shifts inward. Figure 12-10 shows the
effect of this IS shift graphically.
Income, interest rates, and consumption all fall, while investment rises. Income falls because at every level of the interest rate, planned expenditure falls. The interest rate falls because the fall in income reduces demand for money; since the supply of money is unchanged, the interest rate must fall to restore money-market equilibrium. Consumption falls both because of the shift in the consumption function and because income falls. Investment rises because of the lower interest rates and partially offsets the effect on output of the fall in consumption. If the Federal Reserve wants to keep output constant, then they must increase the money supply in order to reduce the interest rate and increase output back to its original level. The increase in the money supply will shift the LM curve down and to the right. Output will remain at its original level, consumption will be lower, investment will be higher, and interest rates will be lower.

d. The appointment of a new “dovish” Federal Reserve chairman increases expected inflation.

An increase in expected inflation will reduce people’s demand for money. This fall in the demand for money will shift the LM curve down and to the right, as illustrated in Figure 12-11.

The interest rate will have to fall to maintain equilibrium in the money market and this fall in the interest rate will increase investment spending. Equilibrium output will rise, and so will
consumption. If the Federal Reserve wants to keep output constant, then it must decrease the money supply. The LM curve will shift back to its old position and nothing will change.

4) Consider the economy of Slugikistan.

a. The consumption function is given by $C = 200 + .75(Y - T)$. The investment function is $I = 200 - 25r$. Government purchases and taxes are both 100. For this economy, graph the IS curve for $r$ ranging from 0 to 8.

- The IS curve is given by: $Y = C(Y - T) + I(r) + G$. We can plug in the consumption and investment functions and values for $G$ and $T$ as given in the question and then rearrange to solve for the IS curve for this economy:

\[
Y = 200 + 0.75(Y - 100) + 200 - 25r + 100 \\
Y - 0.75Y = 425 - 25r \\
(1 - 0.75)Y = 425 - 25r \\
Y = (1/0.25)(425 - 25r) \\
Y = 1,700 - 100r
\]

- This IS equation is graphed in Figure 12-12 for $r$ ranging from 0 to 8.

b. The money demand function in Slugikistan is $(\frac{M}{P})^d = Y - 100r$. The money supply $M$ is 1,000 and the price level $P$ is 2. For this economy, graph the LM curve for $r$ ranging from 0 to 8.

- The LM curve is determined by equating the demand for and supply of real money balances. The supply of real balances is $1,000/2 = 500$. Setting this equal to money demand, we find: $500 = Y - 100r$ and $Y = 500 + 100r$. This LM curve is graphed in Figure 12-12 for $r$ ranging from 0 to 8.

c. Find the equilibrium interest rate $r$ and the equilibrium level of income $Y$.

- If we take the price level as given, then the IS and the LM equations give us two equations in two unknowns, $Y$ and $r$. We found the following equations in parts (a) and (b): $IS: Y = 1,700 - 100r$ and $LM: Y = 500 + 100r$. Equating these, we can solve for $r : 1,700 - 100r = 500 + 100r$ or $1,200 = 200r$, and $r = 6$. Now that we know $r$, we can solve for $Y$ by substituting it into either the IS or the LM equation. We find $Y = 1,100$. Therefore, the equilibrium interest rate is 6 percent and the equilibrium level of output is 1,100, as depicted in Figure 12-12.
d. Suppose that government purchases are raised from 100 to 150. How does the IS curve shift? What are the new equilibrium interest rate and level of income?

- If government purchases increase from 100 to 150, then the IS equation becomes: \( Y = 200 + 0.75(Y - 100) + 200 - 25r + 150 \). Simplifying, we find: \( Y = 1,900 - 100r \). This IS curve is graphed as \( IS_2 \) in Figure 12-13. We see that the IS curve shifts to the right by 200.

![Figure 12-13](image)

- By equating the new IS curve with the LM curve derived in part (b), we can solve for the new equilibrium interest rate: \( 1,900 - 100r = 500 + 100r \) or \( 1,400 = 200r \), and \( 7 = r \). We can now substitute \( r \) into either the IS or the LM equation to find the new level of output. We find \( Y = 1,200 \). Therefore, the increase in government purchases causes the equilibrium interest rate to rise from 6 percent to 7 percent, while output increases from 1,100 to 1,200. This is depicted in Figure 12-13.

e. Suppose instead that the supply is raised from 1,000 to 1,200. How much does the LM curve shift? What are the new equilibrium interest rate and level of income?

- If the money supply increases from 1,000 to 1,200, then the LM equation becomes: \( (1,200/2) = Y - 100r \), or \( Y = 600 + 100r \). This LM curve is graphed as \( LM_2 \) in Figure 12-14. We see that the LM curve shifts to the right by 100 because of the increase in real money balances.

![Figure 12-14](image)

- To determine the new equilibrium interest rate and level of output, equate the IS curve from part (a) with the new LM curve derived above: \( 1,700 - 100r = 600 + 100r \) or \( 1,100 = 200r \), and \( 5.5 = r \).
Substituting this into either the IS or the LM equation, we find $Y = 1,150$. Therefore, the increase in the money supply causes the interest rate to fall from 6 percent to 5.5 percent, while output increases from 1,100 to 1,150. This is depicted in Figure 1214.

f. With the initial values for monetary and fiscal policy, suppose that the price level rises from 2 to 4. What happens? What are the new equilibrium interest rate and level of income?

i. If the price level rises from 2 to 4, then real money balances fall from 500 to $1,000/4 = 250$. The LM equation becomes: $Y = 250 + 100r$. As shown in Figure 12-15, the LM curve shifts to the left by 250 because the increase in the price level reduces real money balances.